

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Classical Mechanics

Subject Code: 5SC04CLM1

Branch: M.Sc. (Mathematics)

Semester: 4

Date: 22/04/2019

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
 - (2) Instructions written on main answer book are strictly to be obeyed.
 - (3) Draw neat diagrams and figures (if necessary) at right places.
 - (4) Assume suitable data if needed.
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SECTION – I

- Q-1** **Attempt the Following questions** **(07)**
- a. Define: Linear momentum. **1**
 - b. What is virtual displacement? **1**
 - c. Define : Kinetic energy **1**
 - d. For a mechanical system of 3^n particle if there are 3^{n+1} constraints then find degree of freedom. **1**
 - e. Define: Constraint. **1**
 - f. Write the difference between Lagrangian and Hamiltonian of mechanical system. **1**
 - g. What do you mean by ignorable coordinates? **1**
- Q-2** **Attempt all questions** **(14)**
- a. What is degree of freedom? Explain the types of constraints. **8**
 - b. State and prove D'Alembert's principle. **6**
- OR**
- Q-2** **Attempt all questions** **(14)**
- a. State and prove conservation theorem of linear momentum. **4**
 - b. State and prove Lagrange's equation of motion. **10**
- Q-3** **Attempt all questions** **(14)**
- a. Show that the bead sliding on a uniform rotating wire follows the exponential curve. **7**
 - b. Show that Lagrange's equation of motion is invariant under co-ordinate transformation. **7**

OR



- Q-3 Attempt all questions (14)**
- a. Prove that the Lagrange's equation of motion for Atwood's machine is $\ddot{x} = \frac{m_1 - m_2}{m_1 + m_2} g$. 7
- b. State and prove conservation theorem of angular momentum. 7

SECTION – II

- Q-4 Attempt the Following (07)**
- a. What do you mean by homogenous space? 1
- b. What is dual transformation? 1
- c. Write the Lagrange's equation of motion for simple pendulum. 1
- d. Define: Phase space. 1
- e. What can you say about H, if L does not depends on time explicitly? 1
- f. If the system is of 100 particle in 3D with holonomic constraints $y_5 + y_6 = 0, y_1 + y_{100} = 2, y_{11} + y_{22} = 5, y_i + y_j = 3, 20 \leq i, j \leq 30$. then calculate degree of freedom. 2

- Q-5 Attempt all questions (14)**
- a. State Hamilton's principle of motion and hence prove it from Lagrange's equation of motion. 8
- b. Write any three properties of Hamiltonian function also justify it. 6

OR

- Q-5 Attempt all questions (14)**
- a. State and prove Hamilton's equation of motion. 7
- b. Find Hamiltonian and Hamilton's equation of motion for simple pendulum. 7
Find Hamiltonian for the following Lagrangian. Hamilton's equation of motion

$$L = ax^2 + byyx + cx\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{\dot{x}^2 + \dot{y}^2}$$

- Q-6 Attempt all questions (14)**
- a. State and prove the necessary and sufficient condition for canonical transformation. 7
- b. Show that the transformation $Q = \log\left(\frac{\sin p}{q}\right)$ and $P = q \cot p$ is canonical transformation and hence find generating function. 7

OR

- Q-6 Attempt all Questions**
- a. State and prove the matrix form of Hamiltonian equation of motion. 7
- b. State and prove Fermat's Postulate in optics. 7

